Side-Channel Attacks on the McEliece and Niederreiter Public-Key Cryptosystems

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25th of February, 2011

Outline

1) A short introduction to the McEliece PKC

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2) Timing related leakage

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3) Leaking the Goppa polynomial

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4) Leakage from the constant weight encoder

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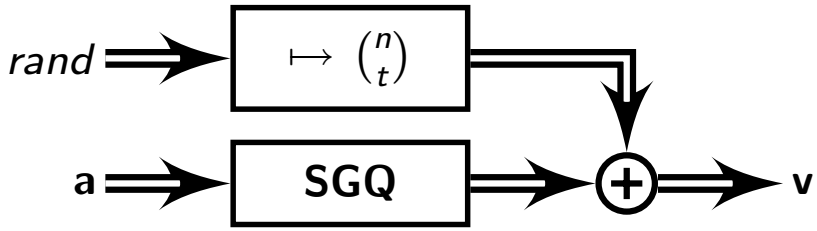
The McEliece PKC I

Encryption:



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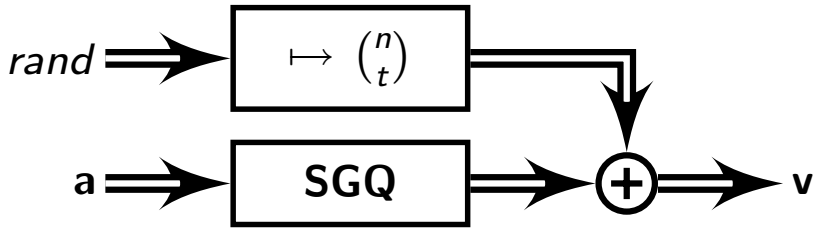
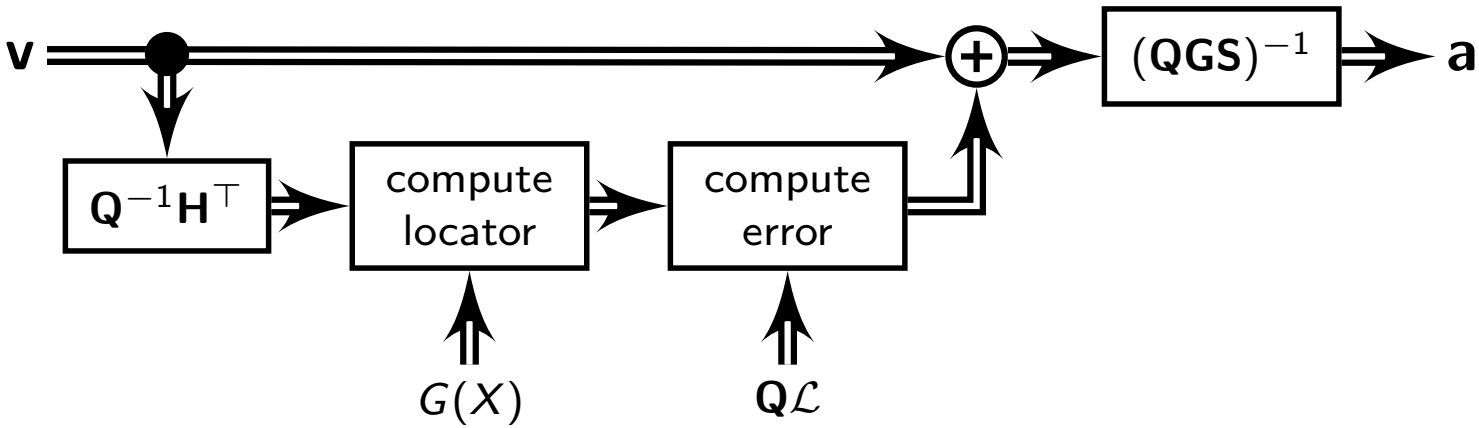


Decryption:



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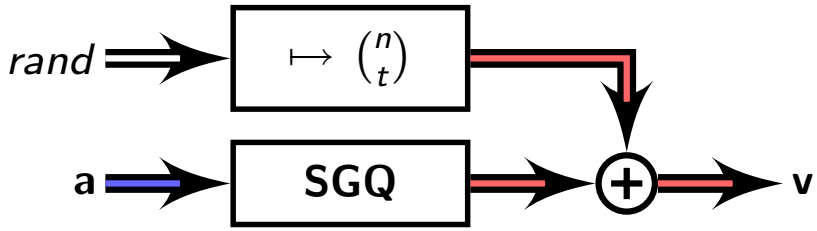

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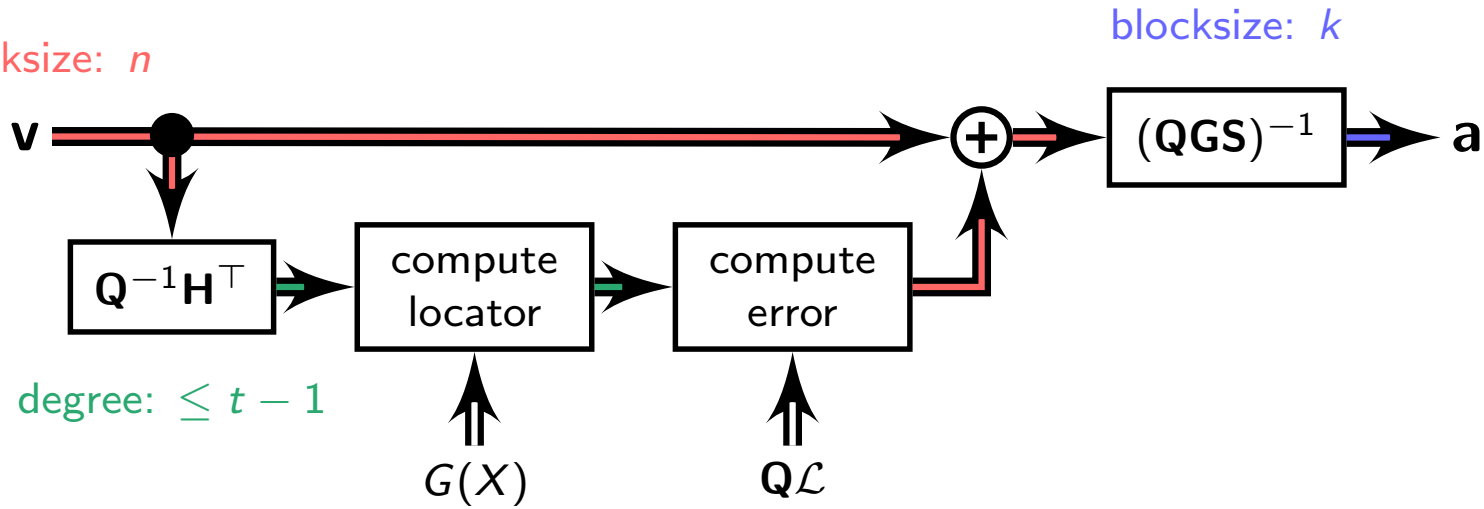

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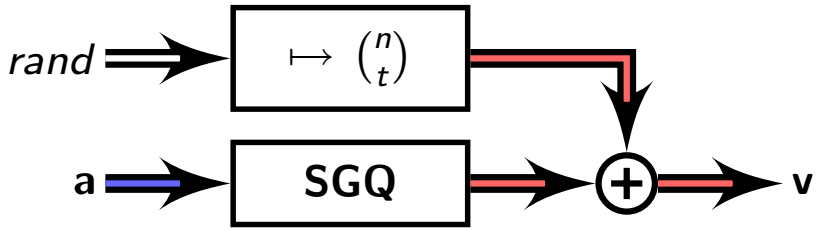
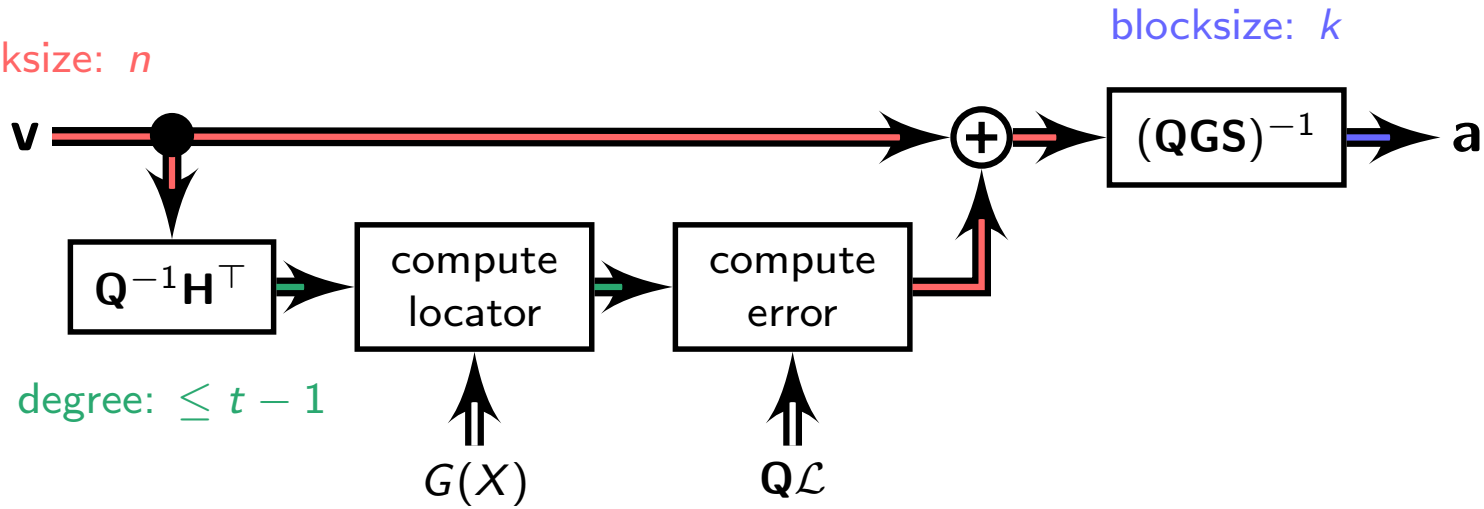
Encryption: $\mathrm{PK}_{\mathrm{McE}} = \{n, k, t, \underbrace{(\mathbf{SGQ})}_{\mathbf{G}'} \in \mathbb{F}_2^{k \times n}\}$



Decryption: $\mathrm{SK}_{\mathrm{McE}} = \{\mathbf{Q}, \mathbf{G}, \mathbf{S}, \mathbf{H}^\top \in \mathbb{F}_{2^m}^{n \times t}, G(X), \mathcal{L}\}$



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Parameters:

Security as in [1]	n	k	t	
original	1024	524	50	insecure!

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- Suitable for post-quantum cryptography
- Adds variety: not based on factorization or DL
- Relatively well researched theory
- Relatively efficient (compared to other PQ PKCs)

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- Otherwise: $w_H(\mathbf{e}) \rightarrow w_H(\mathbf{e}) + 1$
  - $\Rightarrow$ uncorrectable: $\mathbf{c}' \oplus \mathbf{e}'$ instead of $\mathbf{c} \oplus \mathbf{e}$
  - $\Rightarrow$ $w_H(\mathbf{e}') = t$ instead of $w_H(\mathbf{e}) = t + 1$
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- Testing all bits, attacker learns $\mathbf{e}$ using n time measurements
- Attacker is able to remove error and recover data

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Decryption Algorithm - Timing leaks

Algorithm 1 McEliece Decryption based on Patterson's Algorithm

INPUT: A binary ciphertext word $\mathbf{v} \in \mathbb{F}_2^n$.

OUTPUT: The closest cleartext $\mathbf{a}' \in \mathbb{F}_2^k$.

1. $s(X) \leftarrow (\mathbf{v}\mathbf{Q}^{-1}\mathbf{H}^\top)^{-1} \bmod G(X)$ [reported leak 1 [3, 4]]
2. **if** $s(X) = X$ **then** $u(X) \leftarrow s(X)$
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4. $[\mathrm{x}(X), \mathrm{y}(X)] \leftarrow \mathrm{EEA}_{\mathrm{DEC}}(G(X), s'(X), t)$ [reported leak 2 [3, 4]]
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Ignored (so far) timing leak: Cache – LUTs for $\mathbb{F}_{2^m}$.

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Countermeasure I

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- Countermeasure: Assure $\deg(u(X)) = t$ in line 5.
- Problem I: Avoid reaction attack
- Problem II: Avoid inserting roots from $\mathcal{L}$

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Countermeasure II

Our Solution: Add roots from the *non-support* to $u(X)$.

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Definition (Non-Support)

The *non-support* $\overline{\mathcal{L}}$ is defined as

- $\overline{\mathcal{L}} = \mathbb{F}_{2^m} \setminus \mathcal{L}$ iff $\mathcal{L} \subsetneq \mathbb{F}_{2^m}$, as

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In the last case, the error computation has to be done in the extension field $\mathbb{F}_{(2^m)^x}$.

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Countermeasure III

Algorithm 2 Hardened McEliece decryption with Patterson's algorithm.

INPUT: A binary ciphertext word $\mathbf{v} \in \mathbb{F}_2^n$.

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1. ... [Syndrome & locator computation as in alg. 1.]

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1. ... [Syndrome & locator computation as in alg. 1.]
2. $u_h(X) \leftarrow u(X), \quad \overline{u(X)} \leftarrow 1$
3. **for** $i = 0$ **to** $t - 1$
4. $\quad \overline{\gamma_i} \leftarrow$ **chooseElement** $(\overline{\mathcal{L}})$
5. $\quad$ **if** $\deg(u_h(X)) \leq i$ **then** $u_h(X) \leftarrow u_h(X)(X + \overline{\gamma_i})$
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9. $\quad$ **else** $e_i \leftarrow 0$
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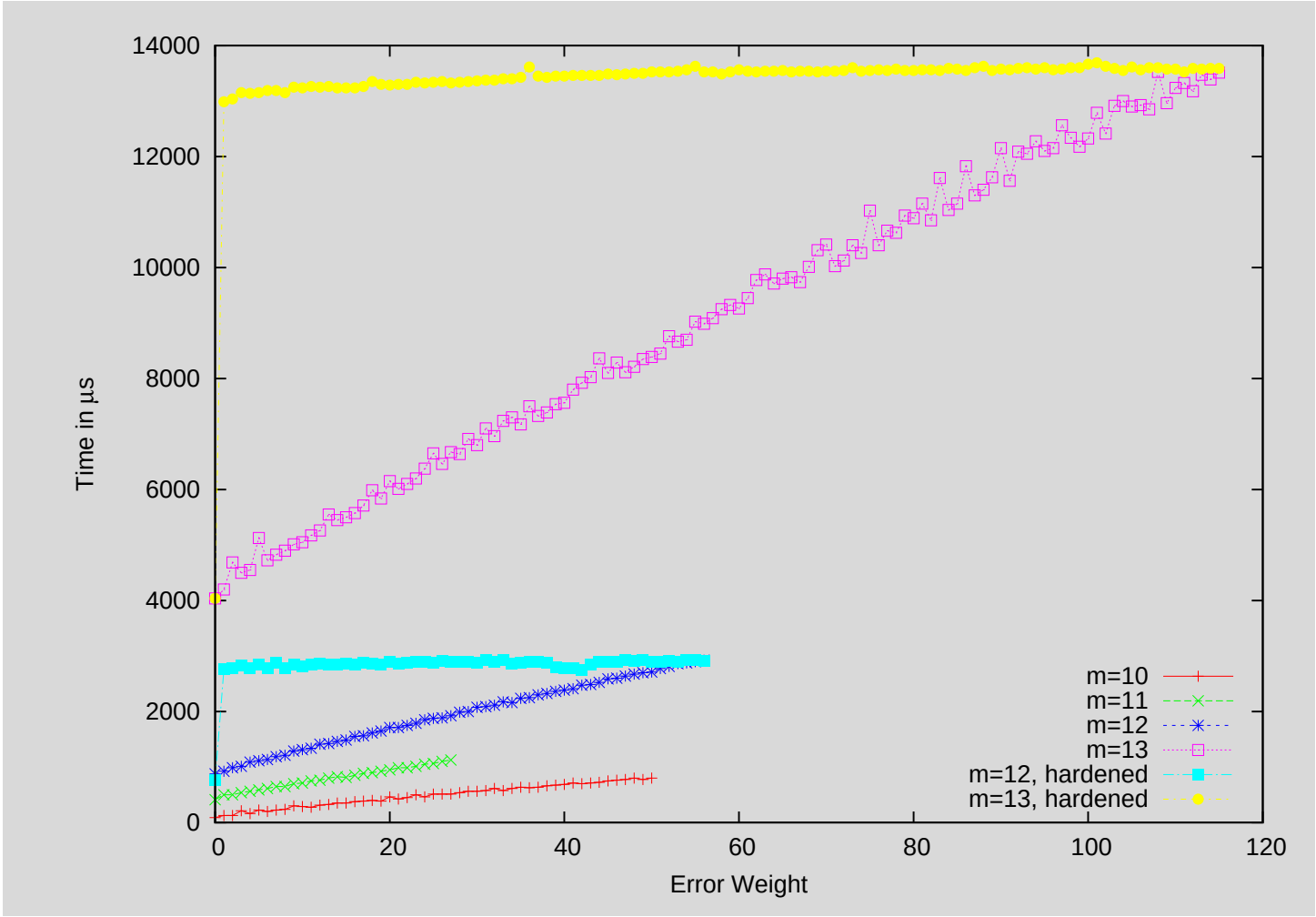
Results



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7. **if** $(u(\gamma_i) = 0)$ **then** $e_i \leftarrow 1$ [FIXED leak]
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$$w_H(\mathbf{e}) \leftarrow \mathit{simple\_but\_unlikely\_function}(\mathbf{v})$$
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- Opinion: Cache leaks more serious (than remaining leaks).

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Leaking the Goppa Polynomial

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- Weak point: Finite field arithmetic
- Finite field arithmetic implemented using lookup tables
  - *"constant"* time multiplications and exponentiations
  - no branches required but:
  - tables need a lot of cache:

$$2.5\text{kB for } m = 10, \ \ldots, \ 26\text{kB for } m = 13$$

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- Steps using $G(X)$:
  - modular inversion (Line 1 of Alg. 1)
  - modular square root computation (Line 3 of Alg. 1)
  - $\mathtt{EEA}_{\mathtt{DEC}}$ (Line 4 of Alg. 1)

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Modular Square-Root Computation

$$\sqrt{s(X)+X} \bmod G(X) \equiv \left(\sqrt{\tilde{s}_1(X)} + \underbrace{\sqrt{X} \bmod G(X)}_{\text{constant}} \sqrt{\tilde{s}_2(X)} \right) \bmod G(X)$$

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   - t relatively small

   $\Rightarrow$ factorization of $z(X)G(X)$ possible.

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$\Rightarrow$ Obtain $G(X)$.

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SPA against the Constant Weight Encoder

- Required for both: McEliece and Niederreiter

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  - Minimum (for our samples): $\approx 13.9\%$
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$\Rightarrow$ **Input could not be reconstructed:**

- too many unknown bits (e.g. $m = 11$: $(1 - 0.266)202$)
- positions of known bits in bit stream unknown

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[5] N. Sendrier. *Encoding Information into Constant Weight Words*. In Proceedings of the 2005 IEEE International Symposium on Information Theory, Adelaide, pages 435-438, Springer-Verlag Berlin Heidelberg, September 2005

Do you have questions?

?**?**?

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25/02/2011

Backup Slides

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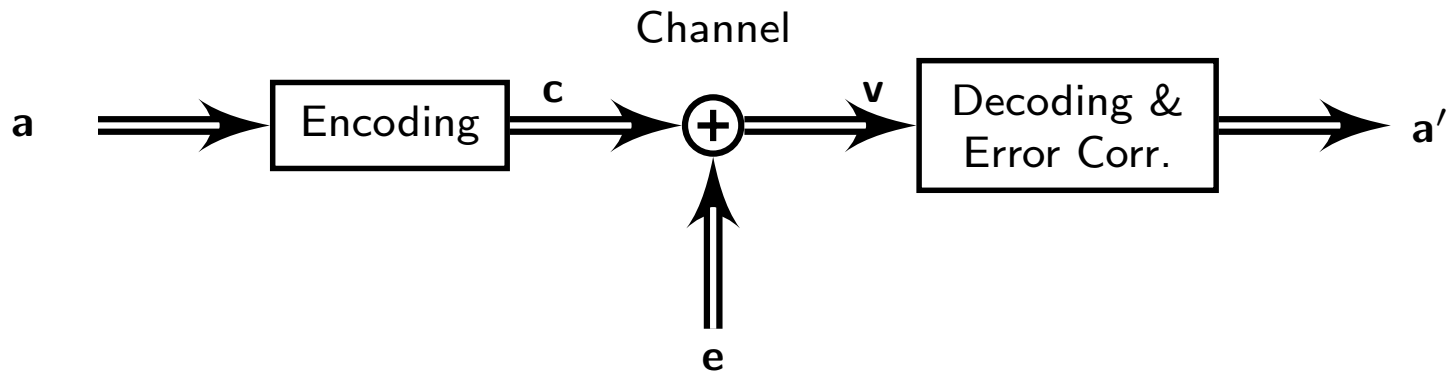
Using Error Correction Codes as PKC



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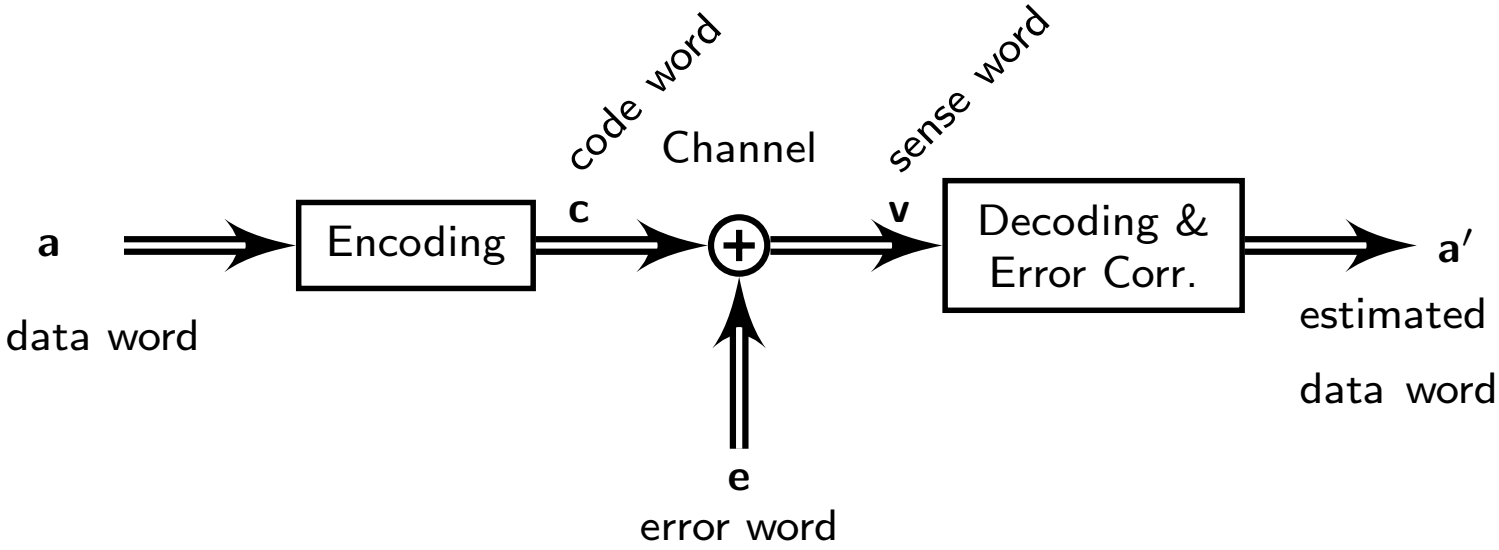

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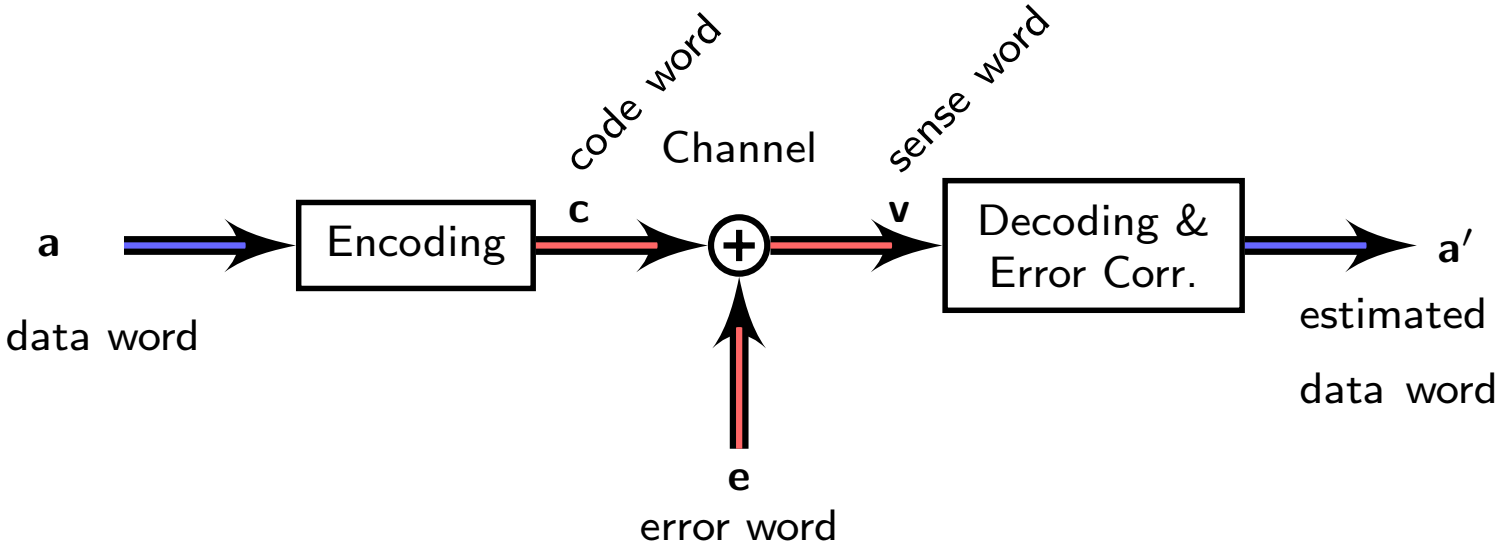

blocksize: k blocksize: n blocksize: k

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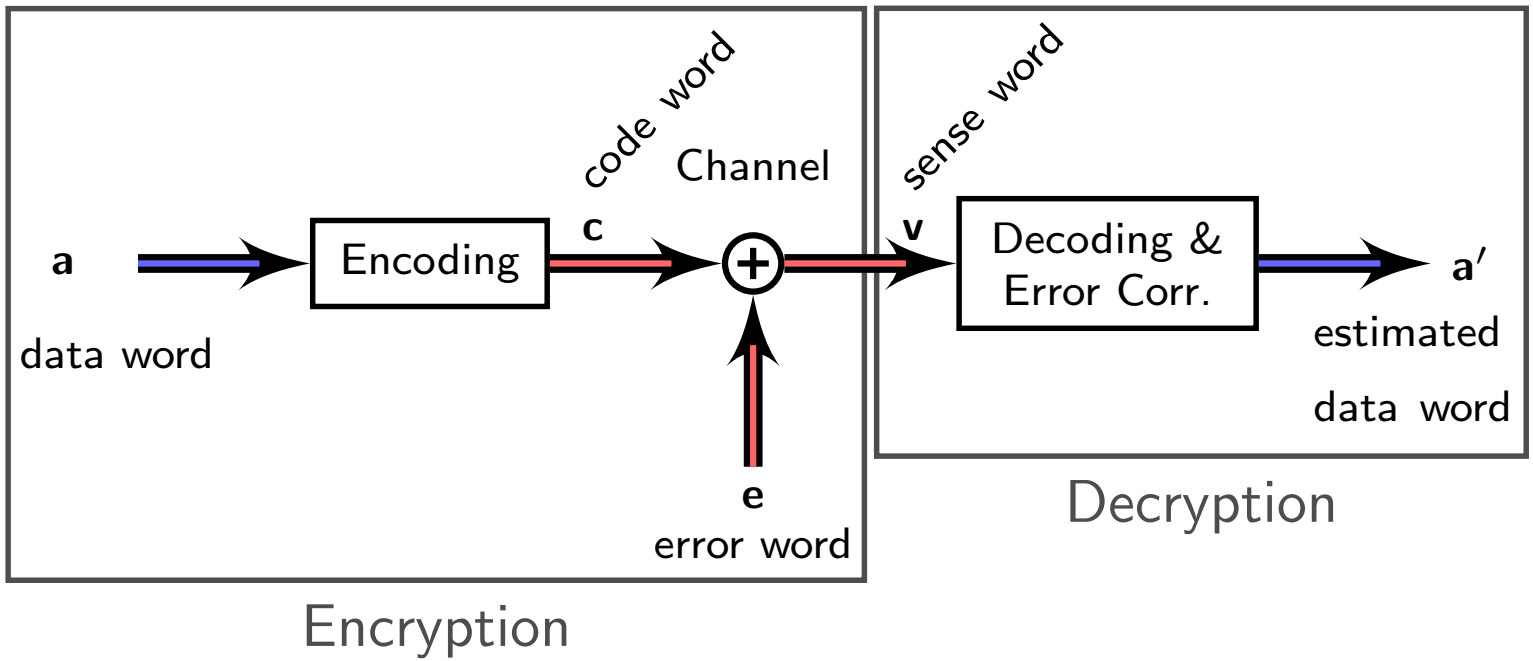
Using Error Correction Codes as PKC

Idea: Error correction for general $(n, k, 2t+1)$ codes is NP-hard



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Decoding & Error Correction for Linear Codes

Classical Goppa Codes (CGCs): subfield codes defined by

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Decoding & Error Correction for Linear Codes

Classical Goppa Codes (CGCs): subfield codes defined by irreducible *"Goppa polynomial"* $G(X)$ with $\deg(G(X)) = t$ and code support $\mathcal{L} \subseteq \mathbb{F}_{2^m}[X]$

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Syndrome Computation with $\mathbf{H} \in \mathbb{F}_{2^m}^{t \times n}$, derived from $G(X)$:

$$\sum_{j=0}^{n-1} \frac{c_j}{X - \gamma_j} \equiv 0 \bmod G(X), \qquad c_j \in \mathbb{F}_2,\ \gamma_j \in \mathcal{L} \subseteq \mathbb{F}_{2^m}$$

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Error Computation find $\gamma_j \in \mathcal{L}$ s.t. $u(\gamma_j) = 0$ for $u(X) \in \mathbb{F}_{2^m}[X]$.

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Decryption Algorithm

Algorithm 1 McEliece Decryption based on Patterson's Algorithm

INPUT: A binary ciphertext word $\mathbf{v} \in \mathbb{F}_2^n$.

OUTPUT: The closest cleartext $\mathbf{a}' \in \mathbb{F}_2^k$.

1. $s(X) \leftarrow (\mathbf{v}\mathbf{Q}^{-1}\mathbf{H}^\top)^{-1} \bmod G(X)$ [Syndrome computation.]

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3. **else** $s'(X) \leftarrow \sqrt{s(X) + X} \bmod G(X)$
4. $[\mathrm{x}(X), \mathrm{y}(X)] \leftarrow \mathrm{EEA}_{\mathrm{DEC}}(G(X), s'(X), t)$ [Break off iff:]
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6. **for** $\gamma_i \in (\mathbf{Q}\mathcal{L})$ [Error computation.]
7. **if** $(u(\gamma_i) = 0)$ **then** $e_i \leftarrow 1$ [Horner Scheme]
8. **else** $e_i \leftarrow 0$
9. **return** $\mathbf{a}' \leftarrow (\mathbf{v} \oplus \mathbf{e})(\mathbf{G}')^{-1}$

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