# Improved Differential Fault Analysis of Trivium

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# Outline

- Motivation
- Description of Trivium
- Differential fault analysis (DFA)
- Algebraic Attack
- Improved (DFA)
- Our results
- Conclusion

# Motivation

## Algebraic Cryptanalysis

Breaking a good cipher should require "as much work as solving a system of simultaneous equations in a large number of unknowns" (Shannon 1949)

# Motivation

## Algebraic Cryptanalysis

Breaking a good cipher should require "as much work as solving a system of simultaneous equations in a large number of unknowns" (Shannon 1949)

## Differential fault analysis

An attacker can induce faults into a system, compare the correct and the faulty behavior, and use this information to retrieve the secret information inside the system

# Description of Trivium

- Hardware oriented stream cipher
- One of the selected eSTREAM project stream ciphers to the final portfolio
- Very fast and simple internal structure
- Secret key $K = (k_1, \ldots, k_{80})$
- Initial vector $IV = (v_1, \ldots, v_{80})$
- Inner state 288-bit stored in 3 shift registers, at time t

$$(a_{t+1}, \ldots, a_{t+93}, b_{t+1}, \ldots, b_{t+84}, c_{t+1}, \ldots, c_{t+111})$$

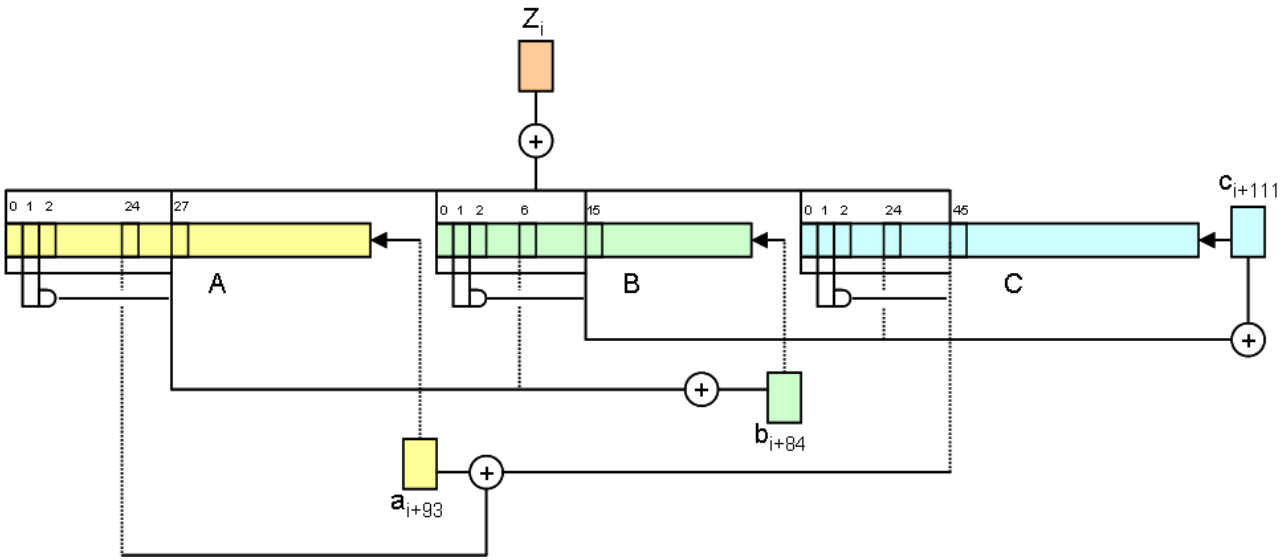
- Initial inner state

$$(\underbrace{0, \ldots, 0, k_{80}, \ldots, k_1}_{A}, \underbrace{0, 0, 0, 0, v_{80}, \ldots, v_1}_{B}, \underbrace{1, 1, 1, 0, \ldots, 0}_{C})$$

# Description of Trivium

## Inner state update

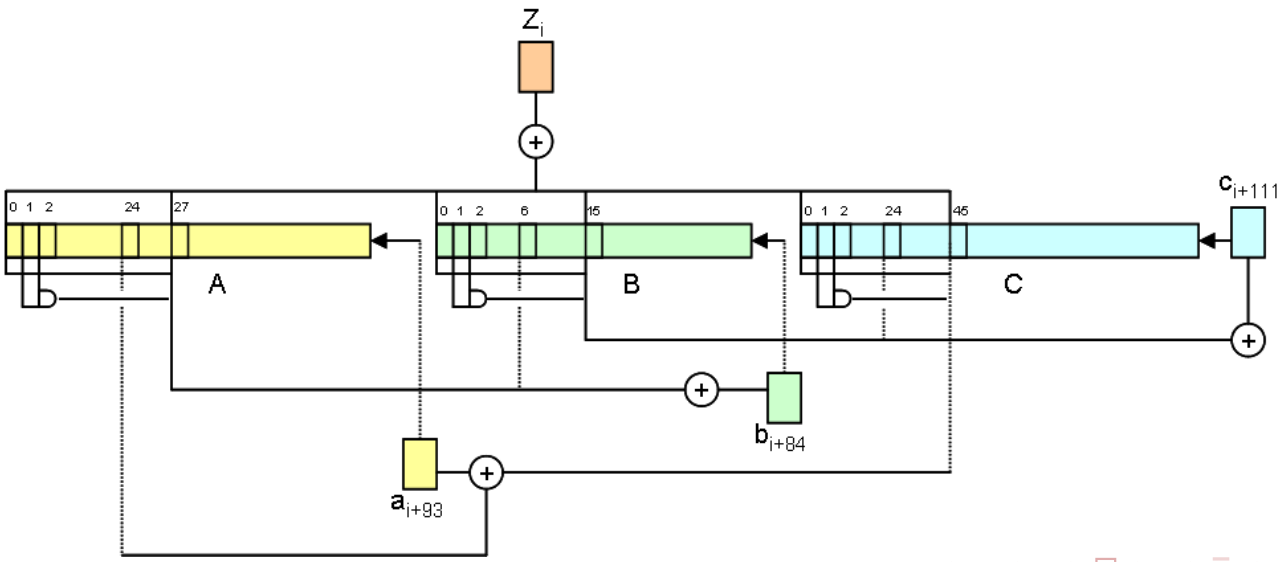
- $a_{i+93} = a_{i+24} + c_{i+45} + c_i + c_{i+1} \cdot c_{i+2}, i \geq 1$
- $b_{i+84} = b_{i+6} + a_{i+27} + a_i + a_{i+1} \cdot a_{i+2}, i \geq 1$
- $c_{i+111} = c_{i+24} + b_{i+15} + b_i + b_{i+1} \cdot b_{i+2}, i \geq 1$



# Description of Trivium

## Output keystream

- Initial phase, Trivium loops 1152 without producing keystream
- After the initial phase, Trivium starts to generate output keystream (time $t = 0$)
- Inner state at $t = 0$, $(a_1, \ldots, a_{93}, b_1, \ldots, b_{84}, c_1, \ldots, c_{111})$
- $z_i = a_i + a_{i+27} + b_i + b_{i+15} + c_i + c_{i+45}, i \geq 1$



# Differential Fault Analysis

## Fault analysis models

- Bit-failures in the data (memory cell)
- Faults affect the implementation code of the device
- Applying some faults to the internal registers of the device
  - Number of faults
  - Location
  - timing
- Attacker can rest the device to its initial state

## Successful attacks

- Fault analysis of stream ciphers (Hoch and Shamir), CHES 2004
- Floating fault analysis of Trivium (Hojsík and Rudolf), IndoCrypt 2008

# Differential Fault Analysis of Trivium

Attacker is able to inject a one-bit fault at a random position within the inner state.

## DFA attack assumption

- Attacker can obtain up to $n$ keystream bits $z_i$ generated by the inner state $(IS_t)$, $0 \leq t \leq n$
- Attacker can inject one-bit fault into $IS_0$
- Attacker can obtain both the cipher output before and after the fault injection $z$ and $z'$
- Attacker can reset the cipher and repeat the previous two steps $m$ times

# Differential Fault Analysis of Trivium

## Attack description

**Require** $m$ fault injection positions $(l_1, \ldots, l_m)$ and the vectors

$$Z(z_1, \ldots, z_n),\ Z^{(i)}(z^{(1)}, \ldots, z^{(n)}),\ 1 \leq i \leq m$$

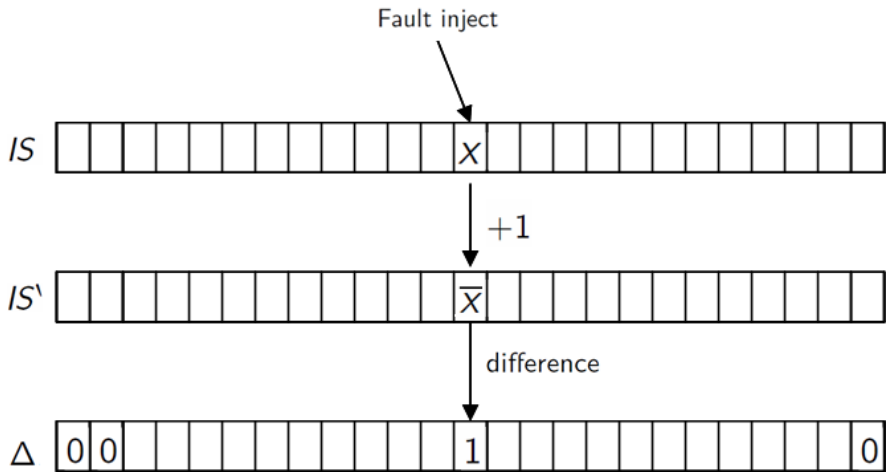
$P \leftarrow \text{EQgenerator}(l_1, \ldots, l_m, Z, Z^{(1)}, \ldots, Z^{(m)})$

$IS \leftarrow \text{extract } (a_1, \ldots, a_{n+93}, b_1, \ldots, b_{n+84}, c_1, \ldots, c_{n+111})$

Recover the secret key $K$ from $IS$

**return** $K$

# Differential Fault Analysis of Trivium



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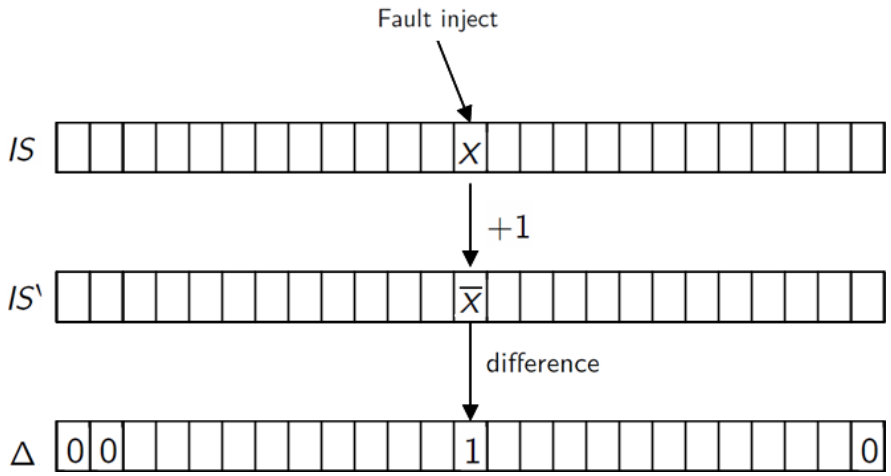


## DFA additional equations

- The DFA equations were initially introduced by Hojsík-Rudolf, IndoCrypt 2008
- $\triangle a_{i+93} = \triangle a_{i+24} + \triangle c_{i+45} + \triangle c_i + \triangle(c_{i+1} \cdot c_{i+2})$
- $\triangle b_{i+84} = \triangle b_{i+6} + \triangle a_{i+27} + \triangle a_i + \triangle(a_{i+1} \cdot a_{i+2})$
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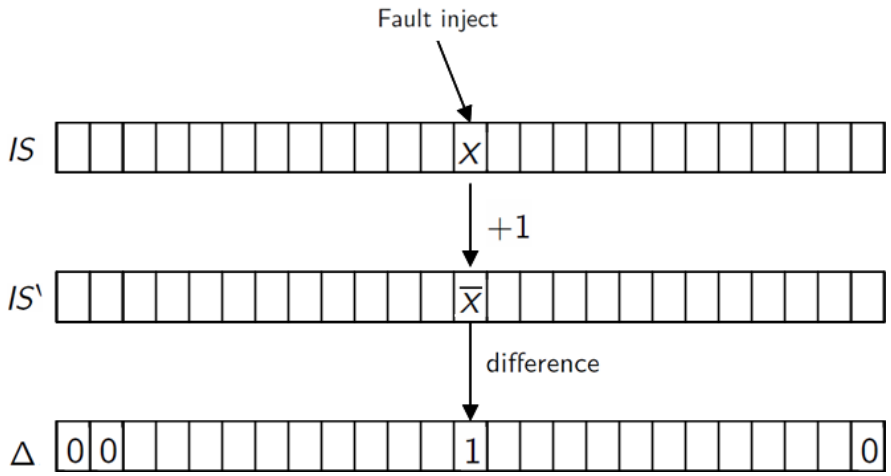


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- $\triangle z_i = \triangle a_i + \triangle a_{i+27} + \triangle b_i + \triangle b_{i+15} + \triangle c_i + \triangle c_{i+45}, \quad i \geq 1$

# Differential Fault Analysis of Trivium

## Equation system describing $n$ keystream bits

**TRIV**$(Z = (z_1, \ldots, z_n))$

**for** $i = 1$ *to* $n$

$$P \leftarrow P \cup \{z_i + a_i + a_{i+27} + b_i + b_{i+15} + c_i + c_{i+45} = 0\}$$

$$P \leftarrow P \cup \{a_{i+93} + a_{i+24} + c_{i+45} + c_i + c_{i+1} \cdot c_{i+2} = 0\}$$

$$P \leftarrow P \cup \{b_{i+84} + b_{i+6} + a_{i+27} + a_i + a_{i+1} \cdot a_{i+2} = 0\}$$

$$P \leftarrow P \cup \{c_{i+111} + c_{i+24} + b_{i+15} + b_i + b_{i+1} \cdot b_{i+2} = 0\}$$

**return** $P$

# Differential Fault Analysis of Trivium

## Generating additional DFA equations

**EQgenerator**$(l_1, \ldots, l_m, Z, Z^{(1)}, \ldots, Z^{(m)})$

$P \leftarrow \mathrm{TRIV}(Z)$

**for** $j = 1$ *to* $m$

$\mathrm{InjectFault}((a_1, \ldots, a_{93}, b_1, \ldots, b_{84}, c_1, \ldots, c_{111}), 1 \leq l_j \leq 288)$

**for** $i = 1$ *to* $n$

$$\triangle a_{i+93} = \triangle a_{i+24} + \triangle c_{i+45} + \triangle c_i + \triangle(c_{i+1} \cdot c_{i+2})$$

$$\triangle b_{i+84} = \triangle b_{i+6} + \triangle a_{i+27} + \triangle a_i + \triangle(a_{i+1} \cdot a_{i+2})$$

$$\triangle c_{i+111} = \triangle c_{i+24} + \triangle b_{i+15} + \triangle b_i + \triangle(b_{i+1} \cdot b_{i+2})$$

$$\triangle z_i \leftarrow z_i + z_i^{(j)}$$

$$P \leftarrow \triangle z_i + \triangle a_i + \triangle a_{i+27} + \triangle b_i + \triangle b_{i+15} + \triangle c_i + \triangle c_{i+45} = 0$$

$S \leftarrow \mathrm{CyclicSubstitute}(P)$

$\mathrm{Substitute}(\triangle A, \triangle B, \triangle C, A, B, C, S)$

**return** $P$

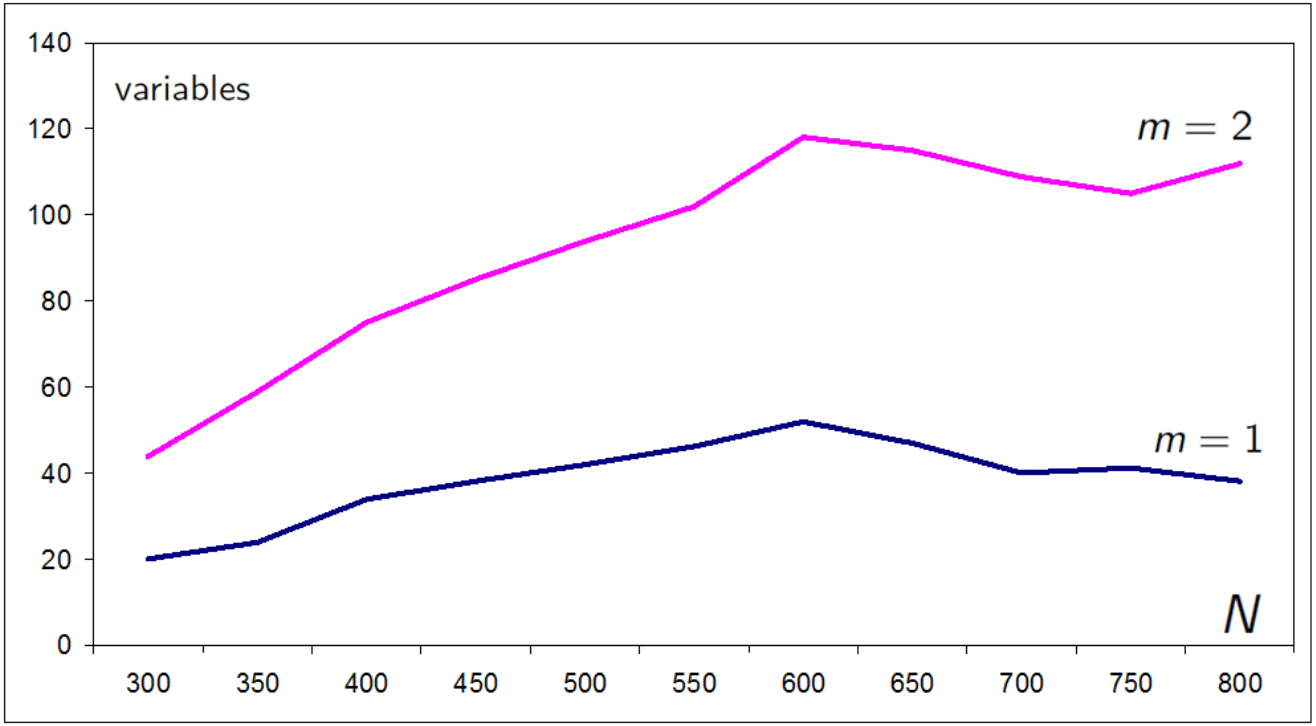
Note: $\triangle(a_{i+1} \cdot a_{i+2}) = \triangle a_{i+1} \cdot a_{i+2} + a_{i+1} \cdot \triangle a_{i+2} + \triangle a_{i+1} \cdot \triangle a_{i+2}$

# Differential Fault Analysis of Trivium

## Equation system specification

| Generator | $m$ | degree 1 | degree 2 | degree 3 | degree 4 |
|---|---|---|---|---|---|
| H-R | 0 | 800 | 2400 | 0 | 0 |
| H-R | 1 | 825 | 2466 | 35 | 57 |
| H-R | 2 | 1017 | 2419 | 36 | 57 |
| H-R | 3 | 1258 | 2298 | 37 | 56 |
| H-R | 4 | 2402 | 498 | 8 | 12 |
| our | 0 | 800 | 2400 | 0 | 0 |
| our | 1 | 994 | 2394 | 28 | 27 |
| our | 2 | 1212 | 2362 | 64 | 76 |
| our | 3 | 1619 | 1990 | 82 | 66 |
| our | 4 | 2688 | 0 | 0 | 0 |

## Number of solved variables



# Algebraic Attacks

## The MP Problem

- Given finite set of multivariate polynomials $P$ in $X = \{x_1, \ldots, x_n\}$ over finite field $F$
- Find $v \in F^n$, $p(v) = 0 \ \forall p \in P$, for example:

$$x_1x_2 + x_1x_3 + x_2x_3 = 0$$
$$x_1x_3 + x_2x_3 + x_1 + 1 = 0$$
$$x_1x_2 + x_1x_3 + x_2x_3 + x_1 + x_2 + 1 = 0$$

- General MP is NP-hard even if $P$ is over $\mathbb{F}_2$ and quadratic
- Hard in average

# Algebraic Attacks

## Attacking Block Cipher

- Using a pair of (known) plaintext-ciphertext values, the secret key and a large number of intermediate variables arising in the cipher
- Solving the resulting multivariate system recovers the secret key

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## Attacking Stream Cipher

- Set up a system of polynomial equations in secret key $K$, intermediate variables $X$ and known keystream bits $z_t$

$$f_1(K, X, Z) = 0, \cdots, f_N(K, X, Z) = 0$$

- Solving the multivariate system to get K

# Algebraic attack on Trivium

## Algebraic attack steps

- $P \leftarrow \text{TRIV}(Z = (z_1, \ldots, z_n))$: return a system of $4n$ equations ($n$ linear and $3n$ quadratic) in $3n + 288$ variables
- $P \leftarrow \text{Solve}(P)$
- Recover the inner state $(a_1, \ldots, a_{93}, b_1, \ldots, b_{84}, c_1, \ldots, c_{111})$
- Clock Trivium backward to recover the secret key $k_1, \ldots, k_{80}$

## Best known algebraic attack on Trivium

- Attacking Bivium using SAT solver ($2^{43}$ s), SAT 2008
- Attacking Bivium using Grain of salt tool ($2^{36.5}$ s), Mate Soos
- Cube attack on scaled version of Trivium (672 and 735 initialization rounds), Dinur and Shamir

# Algebraic Attacks

## Tools for Solving MP

- Computing Gröbner basis
  Gröbner basis algorithms are the best known techniques for solving multivariate systems
- Cube attack
  A method of cryptanalysis applicable to a wide variety of symmetric-key algorithms, published by Dinur and Shamir
- SAT solver-based attacks
  SAT solvers are used in algebraic attacks specially to stream ciphers in recent years

# SAT Solvers

## The Satisfiability Problem

Given a Boolean propositional formula, does there exist assignment of variables such that the formula becomes true

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## SAT Solver-Based Attacks

- Construct a system of equations in the ANF form describing the cipher
- Convert ANF to CNF
- Use a SAT solver to find a solution to the problem
  - MiniSat, PrecoSat, CryptoMiniSat, ...

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## Successful attacks

- Cryptanalysis of stream cipher HiTag2 ($2^{21}$ s), ISC 2009
- Cryptanalysis of Crypto-1 (40 s) using Grain of salt tool, Mate Soos

# SAT Solvers

## ANF to CNF

- A polynomial $p$ in the ANF form is a sum of products (terms) $p = (x \cdot y + x \cdot z + y \cdot z + y \cdot w + z \cdot w + x + z + y + 1)$
- A polynomial $p$ in the CNF form is a set of clauses $(\wedge, \vee, \neg)$
- Replace nonlinear terms with new variables
- $(t = x \cdot y) \equiv (t \Leftrightarrow x \wedge y) \equiv (\overline{b}_1 \vee x) \wedge (\overline{b}_1 \vee y) \wedge (b_1 \vee \overline{x} \vee \overline{y})$
- $t_1 + t_2 \equiv (b_1 \vee \overline{b}_2) \wedge (\overline{b}_1 \vee b_2) \wedge (\overline{b}_1 \vee \overline{b}_2)$
- $1 \equiv (b \vee \overline{b})$
- Cutting number $c$, splitting polynomials into sub-polynomials that have at most $c$ terms, for example let $c = 6$, $p = (x \cdot y + x \cdot z + y \cdot z + y \cdot w, z \cdot w + b),\ b = (x + z + y + 1)$

# Improved DFA of Trivium

## Attack description

**Require** $m$ fault injection positions $(l_1, \ldots, l_m)$ and the vectors

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$P \leftarrow \text{EQgenerator}(l_1, \ldots, l_m, Z, Z^{(1)}, \ldots, Z^{(m)})$

$\text{CNF}(P) \leftarrow$ Converting $P$ to a satisfiability problem in the CNF form

Solution $\leftarrow$ Solve the satisfiability problem $\text{CNF}(P)$ by using MiniSat2

$IS \leftarrow$ extract $(a_1, \ldots, a_{n+93}, b_1, \ldots, b_{n+84}, c_1, \ldots, c_{n+111})$

Recover the secret key $K$ from $IS$

**return** $K$

# Improved DFA of Trivium

## Results of Hojsík-Rudolf

For $N = 800$, the attacker succeeds with probability:

- 2% when $m = 2$ , 78.5% when $m = 3$, 99.8% when $m = 4$
- 100% when $m \geq 5$

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## Our results

For $N \geq 420$, the attacker succeeds with probability:

- 100% when $m = 2$

# Improved DFA of Trivium

## Results of using MiniSat2

| $n$ | $m$ | degree 1 | degree 2 | time (sec.) |
|---|---|---|---|---|
| 800 | 2 | 1216 | 2365 | 0.261 |
| 700 | 2 | 1088 | 2080 | 0.356 |
| 600 | 2 | 1005 | 1771 | 0.414 |
| 500 | 2 | 890 | 1437 | 0.127 |
| 450 | 2 | 831 | 1230 | 1.573 |
| 430 | 2 | 799 | 1138 | 1.936 |
| 420 | 2 | 769 | 1117 | 138.653 |

# Conclusion and Future work

- We used advanced algebraic method (SAT solver) to improve the DFA of Trivium
- Recover the secret key 100% using two fault injections and 420 keystream output bits
- Improve our attack to recover the secret key of Trivium using only one fault injection by applying more advanced conversion techniques and tuning SAT solver parameters

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Thanks for your attention!